CS649
Sensor Networks
IP Track Lecture 3: Target/Source Localization in Sensor Networks

I-Jeng Wang
http://hinrg.cs.jhu.edu/wsn06/

Target/Source Localization in Wireless Sensor Networks

- Basic Problem Statement: Collaborative estimation of the locations of targets (or source signals) based on distributed sensor measurements

- Key distinctions from the self localization problem
  - Non-cooperative sources
  - Uncertainty with source/target signals
    - unknown source signal intensity
    - ambiguity created from multiple targets/sources

A Simple Taxonomy of Approaches (our focus in boldface)

- Sensor Types
  - **Passive**: **Acoustic**, Seismic, Magnetic, etc.
  - Active: Radar, Active Sonar, etc
- Basic Physical Measurements
  - Direction of Arrival (DOA)
    - Suitable for narrow band signals
    - Requires either arrays at each sensor or synchronization across sensors
    - Theoretical analysis suggest inferior performance to the other two approaches*
  - Time Delay of Arrival (TDOA)
    - Suitable for broadband signals
    - Requires accurate measurement of relative time delays between sensor nodes
  - **Received Signal Strength (RSS) or Energy**
    - Requires appropriate signal attenuation model

*H. Wang, L. Yip, K. Yao, and D. Estrin, “Lower bounds of localization uncertainty in sensor networks,” *Proceedings of the IEEE ICASSP*, 2004.

Energy Based Acoustic Source Localization* (Signal Source Case)

- Sensor Measurement Model

known: z_i, r_i, β — unknown: S, X

$$z_i = \frac{S}{\|X - r_i\|^{\beta}} + \nu_i \qquad i = 1, \ldots N$$

  - z_i — Measurement (averaged energy) at sensor i
  - S — Source signal energy
  - $X \in \Re^2$ — Source/Target location
  - r_i — Location of sensor i
  - β — Signal attenuation coefficient (for acoustic sources $\beta \cong 2$)
  - ν_i — Additive independent zero-mean Gaussian noise with known variance σ^2 (or more general with $N(\mu_i, \sigma_i^2)$)

- Goal: Estimate source location X from sensor measurements at sensors $\{z_1, z_2, \ldots, z_N\}$

*Adapted from X. Sheng and Y.-H. Hu, "Maximum Likelihood Multiple-Source Localization Using Acoustic Energy Measurements with Wireless Sensor Networks," *IEEE Transaction on Signal Processing*, vol. 53, no. 1, 2005, pp. 44-53.

Maximum Likelihood Estimate of Source/Target Location

- Likelihood Function:

$$f(z_1,\ldots,z_N \mid \sigma^2, S, r_1,\ldots,r_N) \propto \exp\left\{ \frac{-1}{2\sigma^2} \sum_{i=1}^{N} \left[z_i - \frac{S}{\|X - r_i\|^{\beta}} \right]^2 \right\}$$

- MLE is the solution of a nonlinear least square (NLS) problem:

$$\min_{X,S} L(X,S) = \min_{X,S} \sum_{i=1}^{N} \left[z_i - \frac{S}{\|X - r_i\|^{\beta}} \right]^2$$

- We need at least $N \geq 3$ measurements for a unique solution
- Cramer-Rao bound analysis suggests a dense deployment for small estimate variance

Solving the NLS Problem

- Re-write the objective function L with vectors:

$$L(X,S) = \|\mathbf{z} - Hs\|^2 \qquad \mathbf{z} = [z_1, \ldots, z_N]^T$$

$$H = \left[d_1^{-\beta}, \ldots, d_N^{-\beta}\right]^T, d_i = \|X - r_i\|$$

- Any stationary point of $L(X, S)$ satisfies

$$\frac{\partial L(X,S)}{\partial S} = 0$$

$$\Rightarrow \boxed{S = \left(H^T H\right)^{-1} H^T \mathbf{z}}$$

function of X

(LS-S)

$$\nabla_X L(X,S) = 0$$

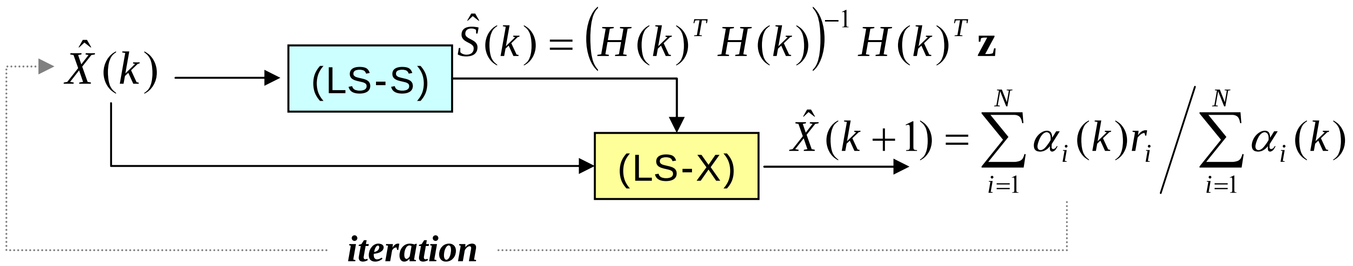
$$\Rightarrow \boxed{X = \frac{\sum_{i=1}^{N} \alpha_i r_i}{\sum_{i=1}^{N} \alpha_i}, \alpha_i = d_i^{-2\beta}\left(z_i - S d_i^{-\beta}\right)}$$

function of X and S

(LS-X)

Two Algorithms for Solving the NLS

- Expectation Minimization (EM) algorithm
  - Starting from an initial estimate of X, $\hat{X}(0)$
  - Repeat until convergence:
    - E-step: Estimate S using (LS-S) with $\hat{X}(k)$ to obtain $\hat{S}(k)$
    - M-step: Update estimate of X using (LS-X) with $\hat{S}(k)$ and $\hat{X}(k)$



- Projection Solution with Search
  - Substitute (LS-S) into $L(X, S)$ to obtain $L'(X)$
  - Apply a multiresolution search algorithm to obtain an estimate of the minimum of $L'(X)$
- Both techniques could be “trapped” in a local minimum

Strength and Issues with the MLE-NLS Based Estimates

- Good:
  - Do not require precise synchronization among sensor nodes
  - MLE is strongly consistent and asymptotically optimal (approaches the CR bound)
  - Experimental results showed significant performance improvement from the simple nearest neighbor localization (location of the sensor with the largest energy)
  - Can be naturally extended to the cases with multiple sources/targets (known number)
- Issues:
  - Require transmission of measurements to a centralized location for processing (sensor selection is a key here*)
  - Issues with local optima and saddle points in NLS
  - Sensitivity to modeling errors (e.g. uncertainties with attenuation parameter, noise characteristics)

*H. Wang, K. Yao, G. Pottie, and D. Estrin, "Entropy-based sensor selection heuristic for target localization," *IPSN'04*, 2004.

Distributed Optimization Algorithms for Solving the NLS*

- A decentralized incremental optimization algorithm:
  - When each sensor receives a new estimate from its neighbor, it makes a small adjustment based on its local measurement z_i and then passes the updated estimate to its neighbors
  - The local update is based on

$$\hat{X}_i(k+1) = \hat{X}_{new}(k) - \alpha \nabla L_i\left(\hat{X}_{new}(k)\right) \quad L_i = \left[z_i - \frac{S}{\left\| X - r_i \right\|^{\beta}} \right]^2$$

new estimate from neighbor

step size > 0

- Convergence to a local minimum (at every sensor) is guaranteed under very mild condition (basically the connectivity among the sensors)
- Theoretically more efficient in terms of energy and bandwidth consumption than centralized approach as the number of nodes increases
- Potential issue with unreliable communications and dynamic connectivity

*M.G. Rabbat and R.D. Nowak "Decentralized source localization and tracking," *Proceedings of IEEE ICASSP*, 2004.

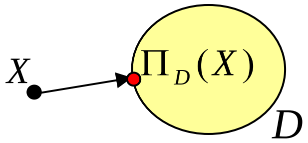
An Alternative Approach

- Projection Onto Convex Sets (POCS) and Aggregated POCS (APOCS)*
  - Formulate the problem as a convex feasibility problem
  - Basic idea: assume S is known

$$\hat{X} \in D = \bigcap_{i=1}^{N} D_i\,,\; D_i = \left\{X \in \Re^2 : \left\| X - r_i \right\| \le \left(S / z_i\right)^{1/\beta}\right\}$$

projection operator

$$\hat{X} = \arg\min_{X} \sum_{i=1}^{N} \left\| X - \Pi_{D_i}(X) \right\|$$



  - A unique solution exists at the true source location if and only if X lies in the convex hull of sensor locations
  - POCS algorithm

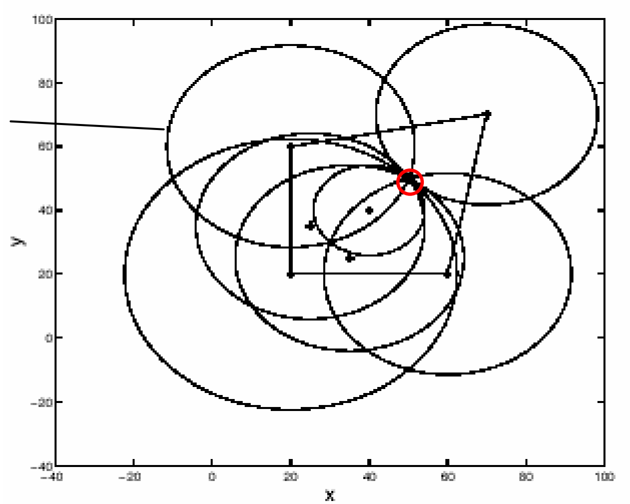
$$\hat{X}(k+1) = \hat{X}(k) + \lambda_k \left[\Pi_{D(k)}\left(\hat{X}(k)\right) - \hat{X}(k)\right]$$

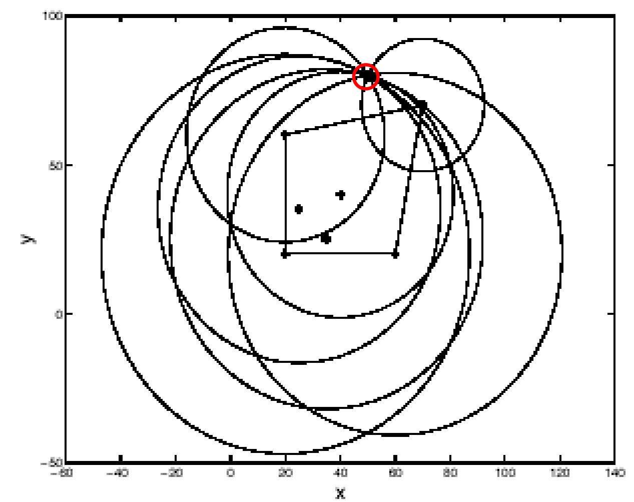
    - $D(k)$ cycles through sensors 1 through N
    - λ_k is a sequence of positive relaxation parameters

*D. Blatt and A.O. Hero, III, "APOCS: A rapidly convergence source localization algorithm for sensor networks," *IEEE ICASSP*, 2005.

Intuitions and Properties

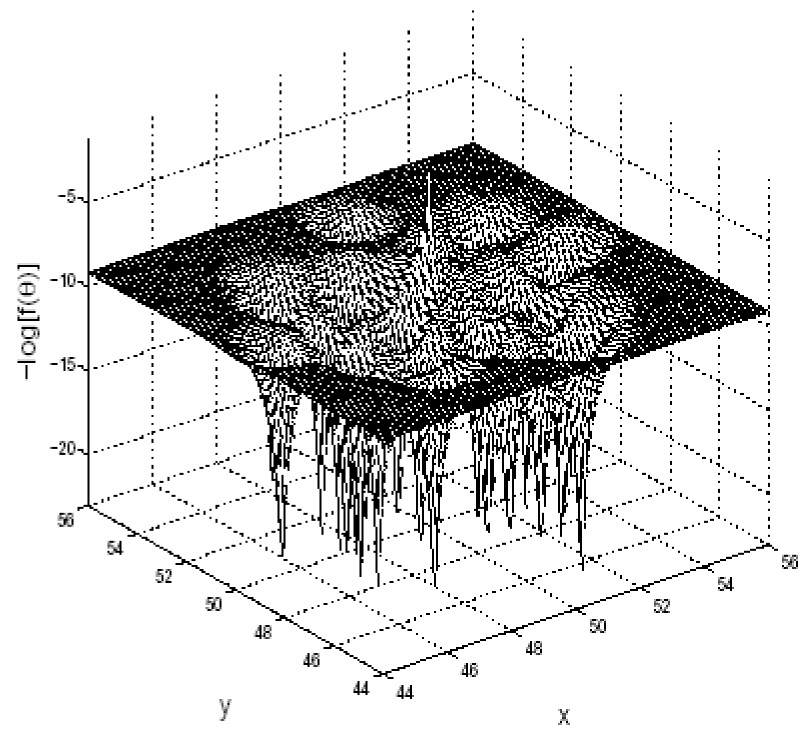
$$z_i = \frac{S}{\|X - r_i\|^{\beta}}$$



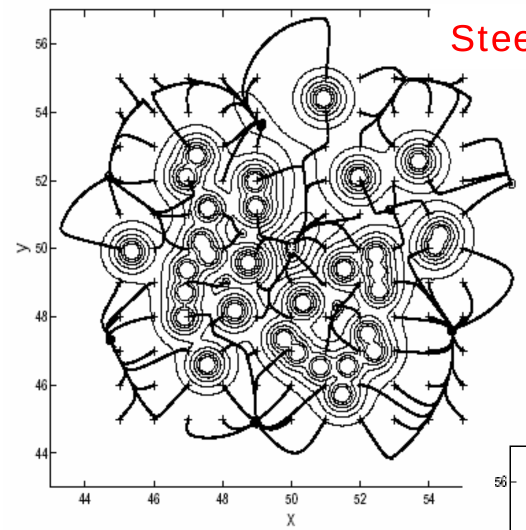


- Distributed implementation possible
- Global convergence is guaranteed under appropriate assumptions
- Require an additional condition on the sensor locations (the convex hull condition)
- Extension to cases with multiple sources and unknown source energy not obvious

Experiment Results for POCS



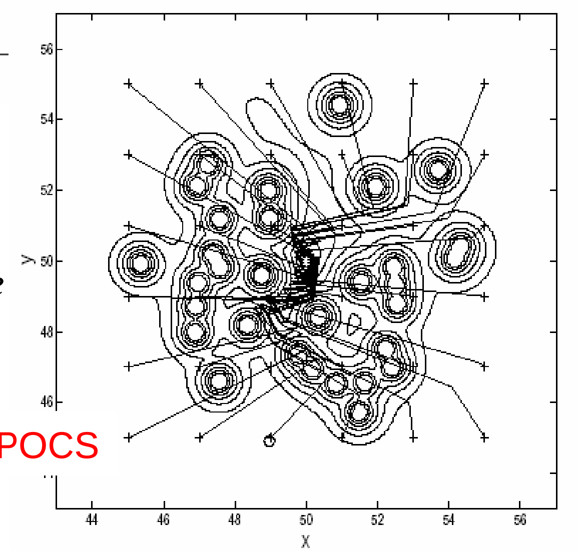
Steepest descent



Convergence to local optima depending on the initial estimate

Global convergence to the vicinity of global optimum

POCS



A Simple Decentralized Averaging Scheme*

- Basic Idea: Estimate the source location by a weighted average of sensor locations (weighted by a function of the RSS at each sensor)

$$\hat{X} = \frac{\sum_{i=1}^{N} r_i g(z_i)}{\sum_{i=1}^{N} g(z_i)}$$

$$g : \Re^+ \rightarrow \Re^+ \text{ monotone increasing}$$

$$g(0) = 0, \lim_{z \to \infty} g(z) < \infty$$

- If the sensor locations are uniformly distributed over a bounded square, then the estimate is unbiased
- g can be selected by minimizing the theoretical asymptotic variance
- Implement the two required averaging using decentralized algorithms (local broadcast, token passing, etc.)
- Experiment results showed better robustness to modeling error in the attenuation parameter than the NLS methods

*M.G. Rabbat, R.D. Nowak, J. Bucklew, “Robust decentralized source localization via averaging,” *Proceedings of IEEE ICASSP*, 2005.