CS649
Sensor Networks
IP Lecture 8: Self-Localization II

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http://hinrg.cs.jhu.edu/wsn06/

Ad-Hoc Localization Using Ranging and Sectoring

K.K. Chintalapudi, A. Dhariwal, R. Govindan,
and G. Sukhatme

IEEE INFOCOM 2004

Summary of Results

- Range-only ad-hoc localization (a small number of anchors) requires high density well beyond the density required for connectivity
  - Require an average degree of 11-12 nodes within the ranging neighborhood to achieve 90% localization and 5% accuracy
- Using both range and bearing information can significantly improve localization performance even with small number of anchors (5%) and low density (average degree at 5)
- Replacing bearing information with sectoring information (60°) can achieve similar performance with 20% anchors and mean degree 5

Three Stages for Localization Schemes

- Estimation distance to anchors
  - Topological approach: DV-dist approach
  - Geometric approach: relative localization (lateration over multiple hops), Euclidean scheme
- Initial position estimate
  - Using multi-lateration to obtain initial position estimate based on ranges to three anchors
- Refinement scheme
  - Perform iterative optimization to refine the position estimates

Iterative Least-Mean Squared Refinement Schemes

- Range-only:

$$\sqrt{(x_i - x_j)^T (x_i - x_j)} - r_{ij} = 0.$$

$$J = \sum_{e_{ij}\in E} \sigma_{ij}^{-2}\left(\sqrt{(x_i-x_j)^T(x_i-x_j)} - r_{ij}\right)^2.$$

$$x_i = \frac{\sum_{n_j\in A_i}\left(\left(\sigma_{ij}^{-2}+\sigma_{ji}^{-2}\right)x_j + \frac{\left(\sigma_{ij}^{-2}r_{ij}+\sigma_{ji}^{-2}r_{ji}\right)(x_i-x_j)}{\sqrt{(x_i-x_j)^T(x_i-x_j)}}\right)}{\sum_{n_j\in A_i}\left(\sigma_{ij}^{-2}+\sigma_{ji}^{-2}\right)},$$

$$\forall n_i \in N - L, n_j \in N.$$

- Range and Bearing:

$$x_i - x_j - \Delta x_{ij} = 0,$$

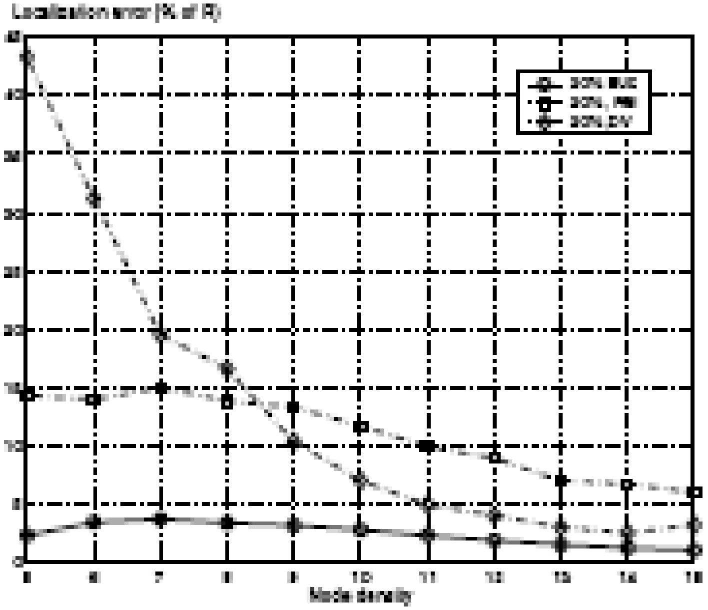
$$J = \sum_{e_{ij}\in E}(x_i-x_j-\Delta x_{ij})^T \Delta M_{ij}^{-1}(x_i-x_j-\Delta x_{ij}).$$

$$x_i = \frac{\left(\sum_{n_j\in A_i}\begin{pmatrix}\Delta M_{ij}^{-1}(x_j+\Delta x_{ij})+\\ \Delta M_{ji}^{-1}(x_j-\Delta x_{ji})\end{pmatrix}\right)}{\left(\sum_{n_j\in A_i}\Delta M_{ij}^{-1}+\Delta M_{ji}^{-1}\right)}$$

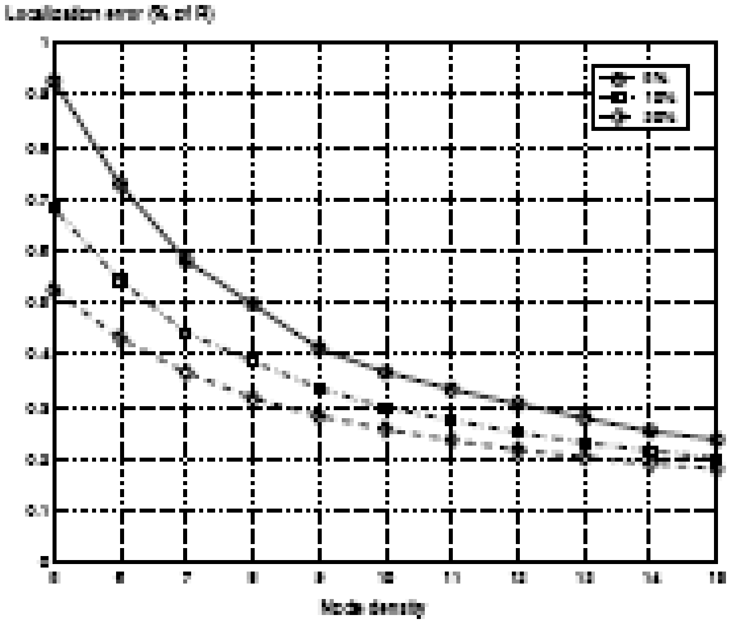
$$\forall i,j | n_i \in N - L, n_j \in N.$$

Experimental Results based on Simulation

- Range error: Assume zero-mean Gaussian noise with $\sigma(\mathbf{r}) = \alpha r$, $\alpha = 0.7$ (derived from sonar ranging data)



Range-only localization error



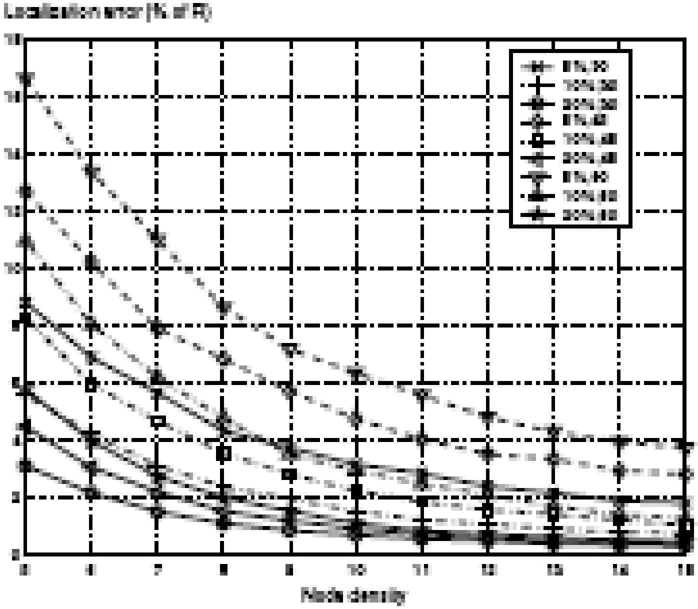
localization error with both range and bearing

Localization with Range and Sector Information

- Using the center of the sector to obtain in initial position estimate based on

$$x_j = x_i + \begin{bmatrix} r_{ij}\cos(\theta_{ij}) \\ r_{ij}\sin(\theta_{ij}) \end{bmatrix}.$$

- Run the refinement scheme based on only range estimates



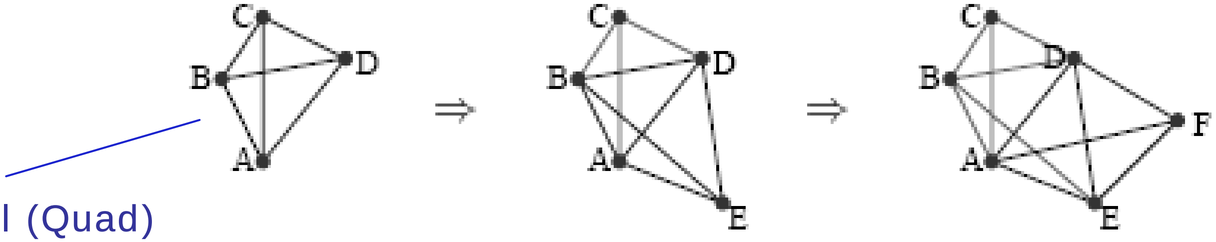
Robust Distributed Network Localization with Noisy Range Measurements

D. Moore, J. Leonard, D. Rus, and Seth Teller

SenSys 2004

Summary

- Explicitly address the impact of measurement errors on localization by studying the robustness of *quadrilateral* localization
- Fully distributed and require no beacons or anchors (focus on relative coordinate)
- Localizes each node correctly with high probability or not at all (prefer accuracy over completeness)
- Cluster-based approach to support dynamic insertion and mobility



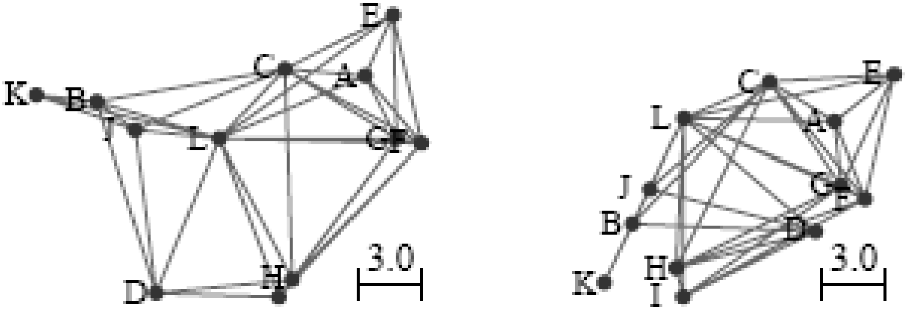
Quadrilateral (Quad)

Challenges with Robust Range-based Localization: Ambiguities in Graph Realization with Noise

- Typical range-based localization techniques focus on minimizing

$$\sigma_{err}^2 = \sum_{i=1}^{M} \frac{(\bar{d}_i - \hat{d}_i)^2}{M}$$

- Graph realization problem: Find the Euclidean positions for the vertices of a graph given the lengths of edges
  - Rigid graphs versus non-rigid graphs

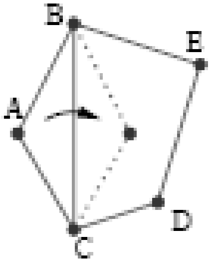


(a) Ground truth (b) Alternate realization

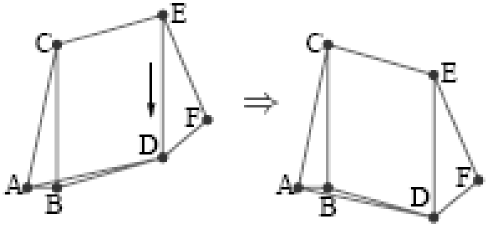
$\sigma_{err} = 0.37$ $\sigma_{err} = 0.34$

Ambiguities with Realization of Rigid Graphs (lead to large localization errors)

- Flip ambiguities



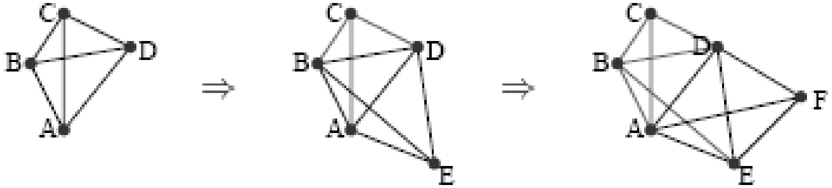
- Discontinuous flex ambiguities



- Measurement noises can lead to graph with these ambiguities
- Proposed approach: look for quadrilaterals that have small probability of ambiguities in presence of measurement errors (*robust quad*)

Three Phases of Algorithms

- Phase I: Cluster Localization
  - A cluster (defined for each node): The node itself (center) and its neighbors (nodes with both communications and ranging capabilities to the center)
  - Localize all robust quads in the cluster



- Phase II: Cluster Optimization (optional)
  - Refine results from phase I by running optimization algorithms
- Phase III: Cluster Transformation

Cluster Localization through Robust Quads

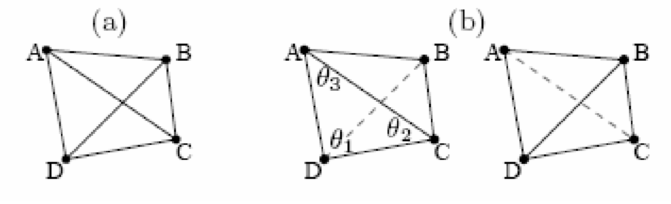
- Robust Quadrilateral: A fully connected graph of four nodes such that every sub-graph of three nodes form a *robust triangle*

threshold based on noise characteristic

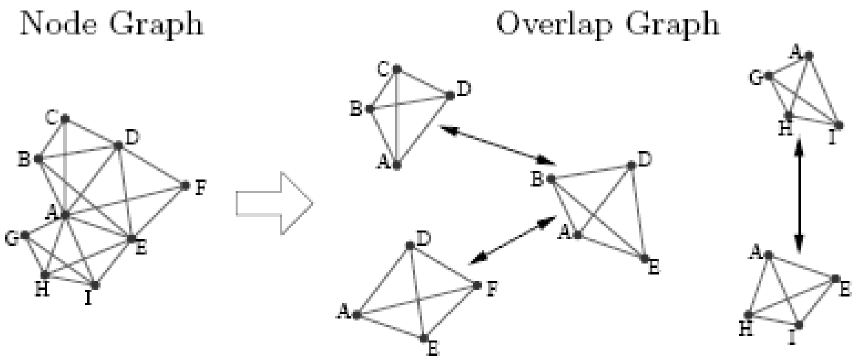
shortest side

$$b\sin^2\theta > d_{\min},$$

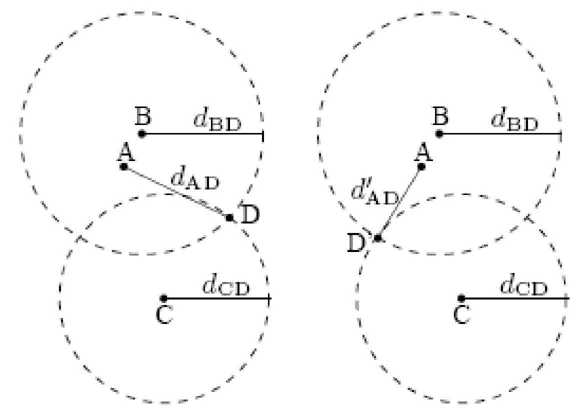
smallest angle



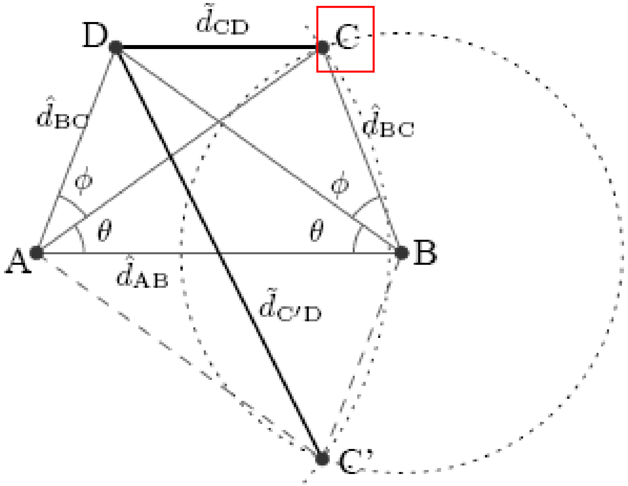
- The probability of error (flip or flex) with a robust quad can be bounded with proper choice of d_{min}
- Only nodes in the largest subgraph of *overlapping* robust quads will be localized



Flip Robustness Analysis (quads have no flex ambiguity based on Laman's Theorem)



A measurement error leads to a flip ambiguity



A flip occurs when range error $\geq \frac{1}{2}(\tilde{d}_{C'D} - \tilde{d}_{CD})$

$$\begin{aligned} d_{err} &= \frac{\tilde{d}_{C'D} - \tilde{d}_{CD}}{2} \\ &= \hat{d}_{AB}\,\frac{\sqrt{\sin^2\phi + 4\sin^2(\theta+\phi)\sin^2\theta} - \sin\phi}{2\sin(2\theta+\phi)}. \end{aligned}$$

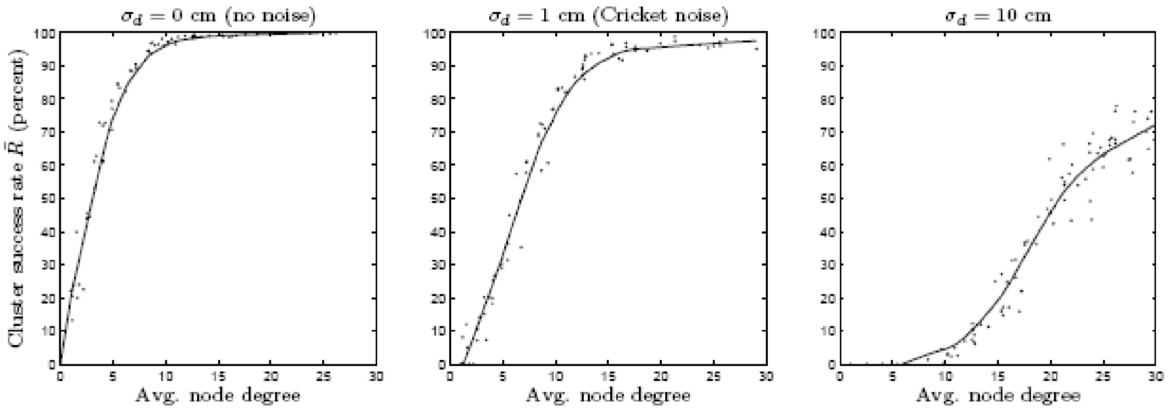
$$min_{\phi} d_{err} = \hat{d}_{AB} \sin^2\theta.$$

If noise is zero-mean Gaussian ⇒ $P(X > d + d_{err}) = \Phi\left(\frac{d_{err}}{\sigma}\right)$

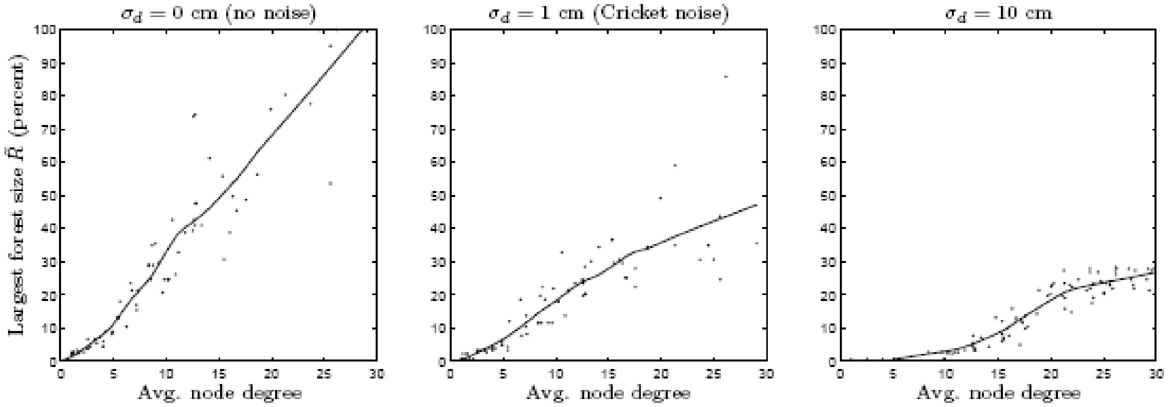
flip probability

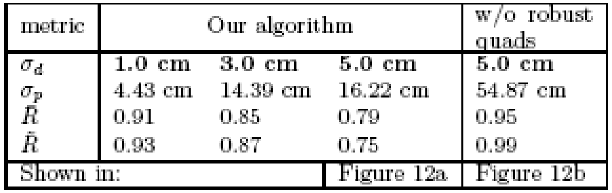
Experimental Results (simulations of a 183 node network)

(a) PLOTS OF CLUSTER SUCCESS RATE, $\bar{R}$, VERSUS NODE DEGREE FOR THE BUILDING ENVIRONMENT



(b) PLOTS OF LARGEST FOREST SIZE, $\tilde{R}$, VERSUS NODE DEGREE FOR THE BUILDING ENVIRONMENT





metric	Our algorithm			w/o robust quads
σ_d	**1.0 cm**	**3.0 cm**	**5.0 cm**	**5.0 cm**
σ_p	4.43 cm	14.39 cm	16.22 cm	54.87 cm
$\bar{R}$	0.91	0.85	0.79	0.95
$\tilde{R}$	0.93	0.87	0.75	0.99
Shown in:			Figure 12a	Figure 12b